Motion control of a novel 6-degree-of-freedom parallel platform based on modified active disturbance rejection controller

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Abstract
A novel 6-degree-of-freedom parallel platform and its motion control based on a modified active disturbance rejection controller are presented in this article. Structure and mathematical model of a novel electromagnetic linear actuator are analyzed, which is the main component of the novel 6-degree-of-freedom parallel platform. Kinematics-based control method of the 6-degree-of-freedom parallel platform is adopted, and the modified active disturbance rejection controller is used to track each desired trajectory accurately in spite of disturbances. Comparative simulations and experimental results demonstrate the effectiveness of the modified active disturbance rejection controller, and this novel 6-degree-of-freedom parallel platform can be driven to realize the given motion well.

Keywords
6-Degree-of-freedom parallel platform, electromagnetic linear actuator, modified active disturbance rejection controller, kinematics-based control, motion control

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Introduction
In modern industry, more and more applications use electromagnetic linear actuators (EMLAs) to achieve high-speed and/or high-precision linear motions, such as robots, manufacturing automation, automobiles, and astronomy.¹² A parallel platform with 6 degrees of freedom (6-DOF) is a kind of parallel robots, which can be driven by various kinds of actuators.³⁴ Compared with conventional actuators such as hydraulic actuators, pneumatic cylinders, and rotary motors together with ball screws,⁵⁶ EMLAs have advantages of fast response, precision motions, and simple structure. However, this structural simplification makes EMLAs easily affected by model uncertainties (e.g. friction, parameter variations, and force ripple) and external disturbances.⁷⁸

Control methods of 6-DOF parallel platforms can be mainly categorized into two types: dynamics-based schemes and kinematics-based schemes.⁹ Accurate dynamic model of the 6-DOF parallel platform is needed in dynamics-based control methods. Errors between the established model and the actual system may reduce control performance. An inverse dynamics controller is implemented in Lee et al.¹⁰ to realize position control of a Stewart platform, and the modeling errors are compensated by a robust controller, which treats the errors as disturbances. A controller–observer scheme is developed in Abdellatif and Heimann¹¹ to control a 6-DOF hexapod robot with a centralized feedforward (FF) dynamics compensation, and a model-based iterative learning controller is designed to further increase the accuracy at high speed. Since dynamic model parameters are time-variant, which change with operation conditions, the calculation in model-based controller is a challenging task.

In contrast, there is no need to know accurate dynamic model in kinematics-based control methods,
which suppose the 6-DOF parallel platform is driven by six independent actuators and makes each actuator track its desired trajectory, respectively. However, six actuators are close coupled and suffering from various disturbances due to the moving load in actual systems. Hence, control performance of the kinematics-based methods greatly depends on the trajectory tracking and disturbance rejection abilities of each axis.

The disturbance observer (DOB) is a good method of compensating disturbances in motion control systems. Nevertheless, since the DOB is designed based on linear system theory, it cannot handle discontinuous disturbances well. Adaptive robust control was proposed to reduce the effect of parametric variations through online parameter adaptation, and uncompensated uncertainties were handled via certain robust control laws. A neural-network-based composite learning controller in Naso et al. and a repetitive model predictive control approach in Cao and Low were designed to suppress reproducible disturbances such as friction and force ripple effectively without accurate models. However, the learning process may take a long time, and they are only applicable to repetitive motions.

We proposed a modified active disturbance rejection controller (MADRC) in Shi and Chang to improve trajectory tracking accuracy of the conventional active disturbance rejection controller (ADRC) besides its excellent disturbance rejection performance by adding a reference acceleration FF. Accurate system model is not required in MADRC, and it is very simple to implement in practice. Furthermore, the MADRC is especially suitable for applications where reference trajectories can be known in advance. In this present article, the MADRC is used to control a novel 6-DOF parallel platform according to kinematics-based control methods. Experimental results indicate that the 6-DOF parallel platform can be controlled well by the MADRC.

Structure analysis of the novel 6-DOF parallel platform

Structure and mathematical model of the novel EMLA

The novel 6-DOF parallel platform is driven by novel EMLAs, which are essentially tubular, moving coil, brushless, and direct current linear motors with axially magnetized permanent magnets for manufacturing simplification consideration. Schematic structure of the EMLA is illustrated in Figure 1, where \( p \) denotes the axis and \( q \) denotes the radius. Since the EMLA is axially symmetrical, only a quarter of it is displayed.

Finite element analysis results indicate that the magnetic field distribution in the air gap is approximately trapezoidal, as shown in Figure 2(a), and the force density is nearly independent of the phase current and mover position, as shown in Figure 2(b).

The dynamics of the EMLA consists of electrical (\( \Sigma i \)) and mechanical (\( \Sigma x \)) parts, which can be described as follows:

\[
\begin{align*}
\Sigma i : \dot{i} & = -\frac{R}{L} i - \frac{K_e}{L} x + \frac{1}{L} u \\
\Sigma x : \ddot{x} & = -\frac{F}{m} + \frac{K_f}{m} i \\
F & = F_f + F_r + F_d
\end{align*}
\]  

where \( u \) is the input phase voltage; \( i \) is the phase current; \( R \) and \( L \) represent the resistance and inductance of the phase, respectively; \( x \) denotes the mover position; \( m \) is the mass of the moving part including load and the coil assembly; \( F \) is the lumped effect of uncertain nonlinearities such as friction \( F_f \), ripple force \( F_r \), and external disturbance \( F_d \); \( K_e \) is the back electromotive force (EMF) coefficient; and \( K_f \) is the electrical–mechanical energy conversion coefficient. Furthermore, in this EMLA, \( K_f \) and \( K_e \) are approximately equal in value.
Components of 6-DOF parallel platform

The 6-DOF parallel platform prototype consists of a base plate, a motion plate, six novel EMLAs, connecting rods, and universal joints, as shown in Figure 3(a), where a body-fixed Cartesian coordinate system is attached to the motion plate.

Rods connect the base and motion plates by universal joints. EMLAs’ linear motion is transferred to the motion plate through connecting rods and universal joints, and then the motion plate can achieve 6-DOF motions.

Axes definition of the motion plate is illustrated in Figure 3(b), and the numbers represent corresponding EMLAs. The attitude sensor used here, with the type of MTi, is produced by Xsens company. The MTi uses inertial sensors in order to estimate the orientation. There is an axes notation printed on the sensor surface, which is shown in Figure 3(c). When installing the attitude sensor, make sure that these two coordinate systems coincide with each other.

The accessorical software of the attitude sensor, MT Manager, is an easy-to-use graphical user interface with possibilities to configure Xsens’ sensors, read out, store,
and show data in real-time charts and visualizations. Experimental angles of the motion plate moving around x-, y-, and z-axes are also acquired by this tool.

**MADRC and precision trajectory tracking control system**

**MADRC details**

Detailed topology of the MADRC is given in Figure 4, which consists of a tracking differentiator (TD), an extended state observer (ESO), a nonlinear proportional–derivative (NPD) controller, and a reference acceleration FF.\(^{18,23}\)

Suppose the plant is a second-order system

\[
\ddot{x} = f(x, \dot{x}, d(t), t) + bu
\]

(2)

where \(x\) is the output to control, \(u\) is the control input, \(b\) is a system parameter, \(t\) is the time, and \(f(x, \dot{x}, d(t), t)\) denotes the total disturbance, which is nonlinear.

The discrete-time form of TD is represented as

\[
\begin{align*}
x_1(k+1) &= x_1(k) + h \cdot x_2(k) \\
x_2(k+1) &= x_2(k) + h \cdot \text{fhan}(x_1(k) - x_d(k), x_2(k), r, h_0)
\end{align*}
\]

(3)

where \(h\) is the sampling period, \(k\) represents the \(k\)th sampling instant, \(x_1\) is a transitional trajectory of the desired signal \(x_d\), \(x_2\) is the differential signal of \(x_1\), and the nonlinear function \(\text{fhan}(x_1(k) - x_d(k), x_2(k), r, h_0)\) is

\[
\text{fhan}(x_1(k) - x_d(k), x_2(k), r, h_0) = \begin{cases} 
-r \cdot \text{sgn}(a), & |a| > d \\
-r \cdot \frac{a}{d}, & |a| \leq d
\end{cases}
\]

(4)

where \(r\) and \(h_0\) are parameters, and \(a\) and \(d\) are determined as follows

\[
\begin{align*}
d &= r \cdot h_0 \\
d_0 &= h_0 \cdot d \\
y_0 &= x_1(k) - x_d(k) + h_0 \cdot x_2(k) \\
a_0 &= \sqrt{d^2 + 8r \cdot |y_0|} \\
a &= \begin{cases} 
x_2(k) + \frac{(a_0 - d_0)}{2} \cdot \text{sgn}(y_0), & |y_0| > d_0 \\
x_2(k) + \frac{a_0}{2d_0}, & |y_0| \leq d_0
\end{cases}
\end{align*}
\]

(5)

Equations (4) and (5) are the important components of conventional ADRC, which are described specifically in Han,\(^{19}\) Sun,\(^{24}\) and Gao.\(^{25}\)

The main role of the ESO is to estimate the total disturbance, and its discrete-time form with the sampling period \(h\) is

\[
\begin{align*}
e &= z_3(k) - x(k) \\
z_1(k+1) &= z_1(k) + h \cdot (z_2(k) - \beta_{03} \cdot e) \\
z_2(k+1) &= z_2(k) + h \cdot (z_3(k) - \beta_{03} \cdot \text{fhan}(e, 0.5, \delta) + b \cdot u(k)) \\
z_3(k+1) &= z_3(k) - h \cdot \beta_{03} \cdot \text{fhan}(e, 0.25, \delta)
\end{align*}
\]

where \(z_1, z_2,\) and \(z_3\) are estimates of the output \(x,\) the derivative of \(x,\) and the total disturbance, respectively.

**Figure 4.** Detailed topology of the MADRC.

MADRC: modified active disturbance rejection controller; NPD: nonlinear proportional–derivative; ESO: extended state observer; TD: tracking differentiator.

\[
\begin{align*}
\beta_{01}, \beta_{02},\text{ and } \beta_{03}\text{ are observer gains, which can be selected as}^{23} \\
\beta_{01} &\approx \frac{1}{h} \\
\beta_{02} &\approx \frac{1}{1.6h^{1/2}} \\
\beta_{03} &\approx \frac{1}{8.6h^{2/3}}
\end{align*}
\]

(7)

The nonlinear function \(\text{fanh}(e, \alpha, \delta)\) is defined as

\[
\text{fanh}(e, \alpha, \delta) = \begin{cases} 
e \cdot \delta^{\alpha-1}, & |e| \leq \delta \\
|e|^\alpha \cdot \text{sgn}(e), & |e| > \delta
\end{cases}
\]

(8)

Since \(z_3\) tracks \(f(x, \dot{x}, d(t), t)\) well in the ESO, the control input \(u\) can be designed as

\[
u = \frac{x_d - z_3}{b}
\]

(9)

to compensate the total disturbance in real time, and therefore, the original nonlinear system (equation (2)) is linearized as

\[
\ddot{x} = \ddot{x}_d
\]

(10)

Then, in theory, the output \(x\) can track the desired signal \(x_d\) accurately. In addition, a NPD controller is added to ensure the actual tracking accuracy in practice.

NPD controller is given as

\[
\begin{align*}
e_1 &= x_1 - z_1 \\
e_2 &= x_2 - z_2 \\
u_0 &= \beta_1 \cdot \text{fanh}(e_1, \alpha_1, \delta) + \beta_2 \cdot \text{fanh}(e_2, \alpha_2, \delta)
\end{align*}
\]

(11)

where \(\beta_1, \beta_2, \alpha_1, \alpha_2,\) and \(\delta\) are controller parameters.

Finally, the control input \(u\) is

\[
u = \frac{x_d - z_3}{b} + u_0
\]

(12)

The robustness properties of conventional ADRC have been mathematically proved in Shi and Chang\(^{18,26}\) and Han,\(^{19}\) and the additional reference acceleration FF in MADRC will not influence the stability of the whole closed-loop system. ESO is a key component of the MADRC, by which the total disturbance can be estimated and compensated in real time, and therefore, the controller becomes robust.
Sufficient conditions of MADRC are listed as follows:

1. Parameter value of \( b \) in equation (2) should be known before designing the controller.

2. \( |f(x, \dot{x}, d(t), t)| \leq M \), \( M \geq 0 \), where \( M \) is the boundary of total disturbance \( f(x, \dot{x}, d(t), t) \).

Moreover, necessary condition of MADRC is \( \lim |z_3 - f(x, \dot{x}, d(t), t)| = 0 \), where \( z_3 \) is the estimate of total disturbance.

**Precision trajectory tracking control system design**

The precision trajectory tracking control system based on the MADRC is illustrated in Figure 5, which consists of current and position loops.

ESO_1 in the current loop is established as

\[
\begin{align*}
    e &= z_{11}(k) - i(k) \\
    z_{11}(k+1) &= z_{11}(k) + h_1 \cdot (z_{12}(k) - \beta_{11} \cdot e + b_1 \cdot u(k)) \\
    z_{12}(k+1) &= z_{12}(k) - h_1 \cdot \beta_{12} \cdot \text{fal}(e, 0.5, \delta_1)
\end{align*}
\] (13)

where \( z_{11} \) is the estimate of \( i \), \( z_{12} \) is the estimate of the total disturbance acting on the electrical subsystem, \( h_1 \) is the sampling period of the current loop, \( \beta_{11} \) and \( \beta_{12} \) are observer parameters, and \( b_1 \) is the system parameter.

NP controller is given as

\[
\begin{align*}
    e_{12} &= u_{11} - z_{11} \\
    u_{01} &= \beta \cdot \text{fal}(e_{12}, 0.5, \delta_1)
\end{align*}
\] (14)

where \( \beta \) and \( \delta_1 \) are controller parameters.

In this trajectory tracking control system, phase currents and the mover position of each EMLA, that is, the leg length, are measured. Attitudes of the motion plate are measured by the gyro, which can also be called the attitude sensor. Hall-effect current sensors and sliding rheostat position sensors are used with accuracy of 0.2% and 0.1%, respectively, and the angle resolution of the attitude sensor is 0.05°.

The input variables of the controller are the desired trajectory \( x_d \), the phase current \( i \) and the actual mover position \( x \) of the actuator, and the controller output at last sampling instant, which is the important difference in MADRC. Phase currents and the mover position are the actuator variables.

**Controller comparisons and experimental setup**

**Comparative simulations**

To test the trajectory tracking and disturbance rejection performance of the MADRC, comparative simulations were implemented on a single EMLA with parameters given in Table 1 using MATLAB/Simulink.

The following three controllers were compared:

1. Proportional–integral–derivative (PID) + FF: PID with reference acceleration FF. Well-tuned PID and proportional–integral (PI) controllers are used in the position loop and the current loop, respectively. Here, classical PID and PI controllers are adopted, which can be designed as follows.

   PID controller is

   \[
   \begin{align*}
   e(k) &= x_d(k) - x(k) \\
   u_1(k) &= K_p \cdot e(k) + K_i \cdot \sum_{j=1}^{k} e(j) \cdot h + K_d \cdot \frac{e(k) - e(k-1)}{h}
   \end{align*}
   \] (15)

   and PI controller is

   \[
   \begin{align*}
   e(k) &= u_1(k) - i(k) \\
   u(k) &= K_p \cdot e(k) + K_i \cdot \sum_{j=1}^{k} e(j) \cdot h_1
   \end{align*}
   \] (16)

   where \( x_d \) is the desired trajectory; \( x \) is the actual mover position; \( i \) is the actual phase current; \( e \) is the error
between desired and actual values; \( h \) and \( h_1 \) are sampling periods of the position loop and the current loop, respectively; \( K_p, K_i, \) and \( K_d \) are controller parameters; \( u_1 \) is the intermediate control variable; and \( u \) is the actual control input.

According to the tuning methods presented in Cominos and Munro, the final controller parameter values are determined by trial and error. PID gains are \( K_p = 34,000, K_i = 1000, K_d = 100, \) and \( h = 0.0002, \) and PI gains are \( K_p = 150, K_i = 10,000, \) and \( h_1 = 0.000025. \)

2. MADRC: Controller parameters in the current loop are as follows: \( h_1 = 0.00025, b_1 = 1/L, b_{11} = 40,000, b_{12} = 5,000,000, \) \( \delta = h_1, \) and \( \beta = 40,000. \) MADRC parameters are as follows: \( h = 0.0002, b = K_i/m \approx 72, \beta_01 = 500, \beta_02 \approx 220,970, \beta_03 \approx 15,967,450, \delta = h = 0.0002, \beta_1 = 34,000, \beta_2 = 100, \alpha_1 = 0.75, \) and \( \alpha_2 = 1.5. \)

3. ADRC: The basic ADRC system is the same as that shown in Figure 5 except for the reference acceleration FF. There is no FF compensation in conventional ADRC, and the controller parameter values are the same as that in MADRC except for \( \beta_1 = 30,000, \beta_2 = 10,000, \) and \( \delta = 25 \times h = 0.005. \)

A desired sinusoidal trajectory is given as \( x_d = 25 \sin (4t - 0.5\pi) + 25 (\text{mm}). \) In addition, a large step external disturbance \( F_d = -100 \text{ N} \) is added at \( t = 0.4 \text{ s} \) and removed at \( t = 1.1 \text{ s} \) to test the performance robustness of each controller to disturbances. Simulation results are compared in Figure 6. It can be seen from Figure 6 that the MADRC achieves good tracking performance in spite of the large external disturbance.

### Experimental setup

To validate the effectiveness of the MADRC when controlling a 6-DOF parallel platform, experimental setup of the control system was constructed, as shown in Figure 7. Sampling periods of current and position loops are 0.025 and 0.2 ms, respectively. dSPACE Autobox is the main controller hardware, and the personal computer (PC) is used to run the application software and display experimental results. Pulse-width modulation (PWM) frequency is set as 40 kHz.

### Results and discussion

Suppose the control object is to make the motion plate move around \( x-, y-, \) and \( z- \) axes simultaneously. The motion equation is

\[
\begin{align*}
\theta_x &= 4.632 + 5.481 \cos (3.749t) + 1.596 \sin (3.749t) (^\circ) \\
\theta_y &= 0.261 - 0.804 \cos (3.749t) - 4.187 \sin (3.749t) (^\circ) \\
\theta_z &= -2.668 - 0.348 \cos (3.749t) + 2.874 \sin (3.749t) (^\circ)
\end{align*}
\]

where \( \theta_x, \theta_y, \) and \( \theta_z \) are angles of the motion plate moving around \( x-, y-, \) and \( z- \) axes, respectively. Since motion equation of the parallel platform is given, desired trajectory of each leg can be obtained by inverse kinematics analysis, and the following are the details.

Desired trajectories of six EMLAs can be calculated by the platform designer, which are given as follows:

### Table 1. Parameters of EMLA.

<table>
<thead>
<tr>
<th>Items</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase resistance (( R ))</td>
<td>3.4 ( \Omega )</td>
</tr>
<tr>
<td>Phase inductance (( L ))</td>
<td>4.42 mH</td>
</tr>
<tr>
<td>Mass of moving part (m)</td>
<td>250 g</td>
</tr>
<tr>
<td>Force constant (( K_f ))</td>
<td>18.01 N/A</td>
</tr>
<tr>
<td>Back EMF constant (( K_e ))</td>
<td>18.01 V/s/m</td>
</tr>
<tr>
<td>Stroke</td>
<td>100 mm</td>
</tr>
</tbody>
</table>

EMF: electromotive force.
\[
\begin{align*}
    x_{d1} &= 20 \sin (5t) + 20 \text{ (mm)} \\
    x_{d2} &= 15 \sin (5t) + 15 \text{ (mm)} \\
    x_{d3} &= 5 \sin (5t) + 5 \text{ (mm)} \\
    x_{d4} &= 10 \sin (5t) + 10 \text{ (mm)} \\
    x_{d5} &= 20 \sin (5t) + 20 \text{ (mm)} \\
    x_{d6} &= 15 \sin (5t) + 15 \text{ (mm)}
\end{align*}
\]

In Su et al., conventional ADRC was used to control a general 6-DOF Stewart platform. Considering the similarity in controller designing method, which is also based on inverse kinematics, ADRC is selected here for experimental comparisons. Experimental results are given in Figures 8 and 9.

It can be seen from Figures 8 and 9 that each EMLA tracks its desired trajectory well, and the motion plate is driven to realize the control object. This indicates the effectiveness of the MADRC when applied to the novel 6-DOF parallel platform.

Since measuring accuracy of each leg position sensor is limited, tracking accuracy of each EMLA in practice is not as high as that in simulations, and consequently, the angle error around each axis of the motion plate exists. Therefore, these errors are mainly caused by low-resolution sensors, and they will not be too much when moving for some more time, which are illustrated in Figure 9(b) for 100 s of operation. Also, this problem can be solved using high-resolution sensors, such as...
as optical linear encoders, which improves the system cost as well.

Moreover, dynamical characteristics of the parallel platform have not been taken into account in this control scheme, and the experimental results will not be as good as that with inverse dynamics–based models, which work for high dynamic loads.

Here, the peak force of this EMLA is designed as 120 N. So the maximum acceleration of the reference trajectory should be limited under this consideration. The peak acceleration levels should also be known so that a specific actuator can be sized according to the force and slew rate demands on it.29

Moreover, for high-frequency motion, the proposed purely kinematics-based scheme cannot handle it perfectly. Some other controllers, such as dynamics-based schemes, may be combined to improve the control performance remarkably.

Since the computation of dynamics models in real-time implementation is time-consuming, simplified inverse dynamics models are presented in Lee et al.10 and Bera et al.30 to improve the computational efficiency.

In Lee et al.,10 the modeling errors are compensated by $H_\infty$ controller to realize position control of a Stewart platform, and in Bera et al.,30 a hybrid position force controller provides robustness against uncertain parameters, disturbances, and unmodelled dynamics. Moreover, it can compensate for large variations in system parameters and can also control the impedance at robot–environment interface. Therefore, the inverse dynamics of parallel platform with robust control can be taken into consideration in our future study.22

In addition, comparisons between several algorithms, which are operated on the 6-DOF platform, are exhibited in Table 2. These controllers are robust nonlinear task space control (RNTC),3 observer–controller with feedforward dynamics (OCFD) compensation,11 nonlinear control with force sensors (NCFS),31 conventional ADRC,6 and the proposed MADRC. It can be seen from Table 2 that the proposed scheme has the
Advantages of both simple computation and high control accuracy.

Conclusion

Motion control of a novel 6-DOF parallel platform based on MADRC is explored in this article. MADRC has the advantages of good trajectory tracking and disturbance rejection performance, which is based on the knowledge of the desired trajectory in advance. Furthermore, for a certain motion of this novel 6-DOF parallel platform, desired trajectories of six EMLAs can be calculated according to kinematics analysis. Good performance of the MADRC depends heavily on advance information of the desired trajectory, and in this 6-DOF parallel platform, desired trajectory of each leg can be obtained in advance theoretically. Moreover, the total disturbance can be estimated and compensated in real time in MADRC, which is the very important performance for controlling the 6-DOF parallel platform. Therefore, the MADRC is especially suitable for controlling the 6-DOF parallel platform theoretically. Comparative simulations and experimental results indicate the effectiveness of the MADRC, and the 6-DOF parallel platform can realize the desired motion well. Consequently, this novel 6-DOF parallel platform along with the MADRC provides a new solution for high-performance motion control applications. Owing to limitations of the leg length, platform workspace, and singularity, the parallel platform design still needs to be optimized. It is our future study to put more effort into the application research of this parallel platform, such as a motion simulator.

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Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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References


**Appendix I**

**Notation**

- \( a, a_0 \): parameters of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( b \): system parameter
- \( b_1 \): system parameter of \( \Sigma \)
- \( d, d_0 \): parameters of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( e, e_1, e_2, e_{12} \): estimating errors
- \( F \): lumped effect of uncertain nonlinearities
- \( F_d \): external disturbance
- \( F_t \): traction
- \( h \): sampling period
- \( h_0 \): parameter of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( i \): sampling period of the current loop
- \( j \): phase current
- \( k \): positive integer
- \( k \): kth sampling instant
- \( K_q \): back EMF coefficient
- \( K_r \): electrical–mechanical energy conversion coefficient
- \( K_{w1}, K_{w2}, K_d \): PID controller gains
- \( L \): phase inductance
- \( m \): mass of the moving part
- \( p \): axial position of EMLA
- \( q \): radius of EMLA
- \( r \): parameter of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( R \): phase resistance
- \( t \): time
- \( u_0, u_1, u_{01} \): intermediate control variables
- \( u \): input phase voltage
- \( x \): mover position
- \( x_1, x_2, x_3, x_0 \): desired signal
- \( x_{d1}, x_{d2}, x_{d3}, x_{d4}, x_{d5}, x_{d6} \): desired trajectories of six EMLAs
- \( x_1, x_2, x_3, x_0, z_1, z_2, z_3, z_{12}, z_{11} \): transitional trajectory of \( x_d \)
- \( \alpha, \alpha_1, \alpha_2 \): differential signal of \( x_1 \)
- \( \beta, \beta_1, \beta_2, \beta_01, \beta_02, \beta_03 \): parameters of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( \delta, \delta_1, \delta_2, \delta_11, \delta_12 \): estimate of the derivative of \( x \)
- \( \theta_x, \theta_y, \theta_z \): estimate of total disturbance
- \( \Sigma \): estimate of total disturbance acting on \( \Sigma \)
- \( \Sigma_i \): parameters of function \( \tan(x_1(k) - x_d(k), x_2(k), r, h_0) \)
- \( \Sigma \): gains of ESO
- \( \Sigma \): gains of ESO in the current loop
- \( \alpha, \alpha_1, \alpha_2 \): angles of the motion plate moving around \( x, y, \) and \( z \)-axes