Theoretical and experimental studies of a switched inertance hydraulic system

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Abstract
A switched inertance hydraulic system uses a fast switching valve to control flow or pressure and is potentially very efficient as it does not rely on dissipation of power by throttling. This article studies its performance using an analytical method which efficiently describes the system in the time domain and frequency domain. A lumped parameter model and a distributed parameter model have been used for investigation using different parameters and conditions. The analytical models have been validated in experiments and the results on a prototype device show a very promising performance. The proposed analytical models are effective for understanding, analysing and optimizing the characteristics and performance of a switched inertance hydraulic system.

Keywords
Digital hydraulics, switched inertance hydraulic systems, switching valve, transmission line model, efficient fluid power

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Introduction
In most fluid power hydraulic systems, speed and/or force of the load are controlled using valves to throttle the flow and thus reduce the hydraulic pressure. This is a simple but extremely inefficient method as the excess energy is lost as heat, and it is common for more than 50% of the input power to be wasted in this way. A switched inertance hydraulic system (SIHS), which performs analogously to an electrical ‘switched inductance’ transformer, is one possible approach to raise efficiency. This technique makes use of the inherent reactive behaviour of hydraulic components. A fluid volume can have a capacitive effect, while a small diameter line can have an inductive effect. This also requires high-speed switching valves to achieve the sufficient switching frequencies and to minimize the pressure and flow loss at the valve orifices. The valve ideally should have low resistance and low leakage and be able to operate with a very fast switching frequency. Numerous valve techniques have been proposed for high-speed switching hydraulic systems, including solenoid valves, poppet valves, linear spool valves and rotary valves. A three-phase rotary alternator valve was designed by Pollard in 1964 for investigating the transmission of power using pulsating flow. The switching valve was used to produce an essentially steady flow. An impedance method was used to study the dynamic responses of pulsating flow in Pollard’s work. However, an analytical solution for the differential equations describing the fluid pipe model and time-variant non-linear valve model in his work is not found. Also, the wave propagation effect is not considered in terms of system efficiency and energy loss. A four-way rotary valve was developed by Brown et al. with a desired switching frequency of 500 Hz. Winkler presented a poppet valve that used multiple metering edges to provide a flow rate of 45 L/min at a pressure drop of just 5 bar. The valve spool was driven by a solenoid with a rise time of approximately 1 ms. Tu et al. described a fluid-driven pulse width modulation (PWM) on/off valve based on a unidirectional rotary spool. The valve was expected to give a flow rate of 40 L/min with a PWM frequency of 84 Hz. A seat-type valve, which switches on and off within 1 ms at a
nominal flow rate of 100 L/min at 5 bar pressure drop, was also developed by Winkler and Scheidl. Kudzma et al.10 proposed a three-way linear-acting high-speed valve which enables low flow resistance (65 L/min at 10 bar), low leakage and very high flow gain. Such high-speed valves give opportunities for improvements in the efficiency of SIHSs.11

Using a high-speed switching valve, the SIHS could potentially provide very significant reduction in power consumption over conventional valve controlled systems. Promising results have been achieved with a SIHS in simulation and experiments.1,3,12-14 These proved that the SIHS concept is viable. Scheidl and Hametner found that the switching frequency is crucial for the energetic efficiency of a SIHS. A detailed study of the efficiency optimal switching frequency related to the resonance system condition was presented in Scheidl and Hametner.15 For the mathematical model of a SIHS, Manhartsgruber et al.16 presented a simulation model of a switching control hydraulic system which comprises a switching valve, a long tube and a hydraulic cylinder. Also a mixed time–frequency-domain modelling method for non-linear hydraulic switching systems has been proposed. The model is effective for estimating system characteristics and efficiency. A similar modelling method was also applied in Wang et al.’s work.17 Comparing the experimental and simulated results, it can be seen that the proposed model is a reliable and accurate method for analysing the detailed dynamic behaviour of a SIHS. However, these models include a set of ordinary differential equations (ODEs) describing the actuator dynamics, a ‘transmission line method’ (TLM) pipeline model and time-variant non-linear valve flow equations which may be difficult and time-consuming to solve. Moreover, these models might only be effective for specific cases due to the parameters used in the simulation and difficult to estimate the general characteristics and trend of the systems. Therefore, an analytical model is desired for understanding and analysing the basic characteristics and performance of a SIHS. De Negri et al.18 developed an analytical model of a SIHS based on a lumped element model and validated in experiments based on a pressure booster system. They successfully described the relationship between the flow rate and pressure in a SIHS, but did not focus on system efficiency and power loss. A mixed time–frequency-domain simulation work of a hydraulic buck converter (HBC) was presented in Scheidl et al.19 The time-domain modelling was applied to the switching valve and check valve and the frequency-domain modelling was applied to the wave propagation in a pipe. The resulting system of non-linear algebraic equations was solved by using a Newton–Raphson method in combination with a smoothing of the non-smooth properties of the check valve. This method was applied in simulation and can be used for a parameter study of the HBC. Therefore, the proposed analytical model should be able to describe the system effectively in equations and easily present the relationship of the switching parameters (switching frequency and ratio), system parameters (valve resistance, tube length and diameter) and system efficiency.

This article develops an analytical model of a three-port SIHS. It helps to understand the physical characteristics of switched devices and gives an effective method to analyse the effect of system parameters. First, a lumped parameter system model is introduced for analysing an ideal switching operation. The derived equations give a clear relationship between the supply pressure, delivery flow rate, flow loss and power loss of a SIHS. These are systematically proposed in this article for the first time. This is followed by a distributed parameter model in the frequency domain which includes the effect of the wave propagation for further investigation of system efficiency and power loss. The dynamic flow rate and pressure are calculated in the frequency domain and transferred to the time domain to analyse the system dynamics. The optimal switching on–off time of the switching valve is validated and system performance is investigated by using the proposed analytical model. Experimental validation on a flow booster test rig is presented and followed by discussions and conclusions.

**Principle and operation**

Two basic modes can be configured by reversing the inlet and outlet connections in a three-port SIHS. In one mode it acts as a flow booster; in the other it performs as a pressure booster.1 Figure 1(a) shows the arrangement of a flow booster. It consists of a high-speed switching valve with one common port, two switched ports and a long, small diameter ‘inertance’ tube. The common port is connected alternately to the high-pressure supply port and then the low-pressure

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**Figure 1.** Schematic diagram of flow booster arrangement: (a) hydraulic circuit and (b) electrical equivalent.
supply port. When one switched port is connected to the common port, the other switched port is closed off.

The switching valve operates cyclically and rapidly such that the high and low pressures are opened alternately. The delivery port might connect to a loading system, and it includes an accumulator or other capacitance such that the load pressure is approximately constant. When the valve is connected to the high-pressure supply port, flow passes from the high-pressure supply to the delivery port and the fluid accelerates in the inertance line. When the valve is open to the low-pressure supply port, fluid is drawn from the low-pressure supply to the delivery port by the momentum of the fluid in the inertance line. The inertance tube in the hydraulic circuit performs the function of the inductor in the electrical circuit which is shown in Figure 1(b). As long as the valve is switched quickly, the delivery flow will only reduce slightly due to a small deceleration of fluid velocity when connected to the low-pressure supply, as shown in Figure 2.

The flow booster can produce a modulated pressure from a fixed pressure supply, behaving like a proportional valve. The flow booster configuration is used for the following discussion. Its characteristics and performance are investigated using a lumped parameter model.

**Lumped parameter model**

The lumped parameter model simplifies the description of the behaviour of the distributed physical system into a system consisting of discrete entities that approximate the behaviour of the distributed system under certain assumptions. For laminar flow in the pipeline, the lumped parameter model takes the form of a series of lumped parameter resistor/inductor/capacitor (RLC) networks. It is a simple method to implement and is very flexible in that variable properties and cavitation can be included. In this section, the lumped model of a switched inertance system was studied with system resistances of the high-speed switching valve and tube, as shown in Figure 3.

The following assumptions are made.

- The high- and low-pressure supply ports of the switching valve have the same resistance $R_v$;
- The resistance $R_v$ is linear and time-invariant;
- Switching occurs instantaneously;
- The supply pressures, $p_H$ and $p_L$, and the delivery pressure $p_d$ are constant;
- There is no valve leakage across the ports.

Using these assumptions, the model in Figure 3 can be described by equation (1)

$$\frac{I}{R} \frac{dq}{dt} + q = \frac{1}{R} \Delta p$$

where $\Delta p = p_H - p_d$ for $0 \leq t \leq \alpha T$, $\Delta p = p_L - p_d$ for $\alpha T < t \leq T$ and $R = R_v + R_i$ is the overall resistance of the switched system.

The inlet pressure $p_i$ can be represented as a periodic waveform whose instantaneous value is constant at $p_H$ with a time period $\alpha T$ and then changes abruptly to $p_L$ for a time $(T - \alpha T)$ as indicated in Figure 4. The corresponding flow rate is plotted in Figure 4.

The equation for the flow rate of the rising portion is an exponential of time constant $\tau = I/R$. The flow rate
would rise to the steady-state value \( q_H = (p_H - p_d)/R \) if the input pressure remained at \( p_H \). If \( q_a \) is the initial value of the flow rate, then the flow rate is given by

\[
q(t) = q_H + (q_a - q_H) \cdot e^{-t/\tau} \quad 0 \leq t \leq \alpha T
\]  
(2)

Similarly, the flow rate for the falling portion is

\[
q(t) = q_L + (q_b - q_L) \cdot e^{-(t-\alpha T)/\tau} \quad \alpha T < t \leq T
\]  
(3)

where \( q_L = (p_L - p_d)/R \), and \( q_b \) is the initial value of the flow rate during the falling phase. Therefore, at the end of the period \( 0 \leq t \leq \alpha T \)

\[
q_b = q(aT) = q_H + (q_a - q_H) \cdot e^{-\alpha T/\tau}
\]  
(4)

At the end of the period \( \alpha T < t \leq T \), under steady conditions the flow rate is the same as at the start of the cycle, \( q(T) = q_a \)

\[
q(T) = q_a = q_L + (q_b - q_L) \cdot e^{-(T-\alpha T)/\tau}
\]  
(5)

Substituting equation (5) into equation (4)

\[
q_b = -q_H \cdot \frac{e^{-\alpha T/\tau}(q_H - q_L) + e^{-T/\tau}q_L}{e^{T/\tau} - 1} + q_a e^{-\alpha T/\tau}
\]  
(6)

\[
q_a = -q_H \cdot \frac{e^{\alpha T/\tau}q_H + e^{T/\tau}q_L}{e^{T/\tau} - 1}
\]  
(7)

Using equations (2) and (7), the flow volume \( V_{ab} \) during the period \( 0 \leq t \leq \alpha T \) is

\[
V_{ab} = \int_0^{aT} q(t)dt = \tau(q_H - q_a)(e^{-\alpha T/\tau} - 1) + q_a \alpha T
\]  
(8)

and the average flow rate from the high-pressure supply port during the period \( T \) is

\[
\bar{q}_H = \frac{V_{ab}}{T} = \frac{\tau(e^{\alpha T/\tau} - e^{T/\tau} + e^{(T-\alpha T)/\tau} - 1)(q_H - q_L)}{(e^{T/\tau} - 1)T} + q_a \alpha T
\]  
(9)

Using equations (3) and (6), the flow volume \( V_{bc} \) during the period \( \alpha T < t \leq T \) is

\[
V_{bc} = \int_{\alpha T}^{T} q(t)dt = \tau(1 - e^{-(T-\alpha T)/\tau})(q_b - q_L) + q_L(T - \alpha T)
\]  
(10)

Thus, the average delivery flow rate is

\[
q_m = \frac{V_{ab} + V_{bc}}{T} = \frac{\tau(q_H - q_a)(e^{-\alpha T/\tau} - 1) + \tau(1 - e^{-(T-\alpha T)/\tau})(q_b - q_L) + q_H \alpha T + q_L(T - \alpha T)}{T}
\]  
(11)

Substituting equations (6) and (7) into equation (11), the following simple relationship can be obtained

\[
q_m = q_H \alpha + q_L(1 - \alpha)
\]  
(12)

Substituting \( q_a = (p_H - p_d)/R \) and \( q_L = (p_L - p_d)/R \) in equation (12)

\[
p_d = p_H \alpha + (1 - \alpha)p_L - q_m R
\]  
(13)

Equation (13) shows the relationship between delivery pressure, supply pressures and delivery flow rate. This shows that with a fixed system resistance, the actual delivery pressure is the difference between the ideal delivery pressure \( p_H \) and the pressure loss due to resistance.

Therefore, the pressure loss can be defined as

\[
p_{\text{loss}} = q_m R
\]  
(14)

Substituting \( q_H = (p_H - p_d)/R \) and \( q_L = (p_L - p_d)/R \) into equations (2) and (3), and using equations (6) and (7), then the flow rate is given by

\[
q(t) = \left(\frac{q_H - p_L}{e^{\alpha T/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau}}{e^{T/\tau} - 1}\right) + q_m \quad 0 \leq t \leq \alpha T
\]  
\[
q(t) = \left(\frac{q_H - p_L}{e^{T/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau} + e^{(T-\alpha T)/\tau}}{e^{T/\tau} - 1}\right) + q_m \quad \alpha T < t \leq T
\]  
(15)

As the previous research stated,\(^{17}\) normally the actual average flow rate from the high-pressure supply port \( q_H \) is slightly higher than the ideal flow rate \( q_{\alpha \beta} \), and the average flow rate from the low-pressure supply port is slightly lower than the ideal flow rate \( q_{\alpha L} \). Thus, system flow loss can be defined as

\[
q_{\text{loss}} = \bar{q}_H - q_{\alpha L}
\]  
(16)

or

\[
q_{\text{loss}} = q_{\alpha L}(1 - \alpha) - \bar{q}_L
\]  
(17)

which represents the extra flow rate supplied from the high-pressure port relative to the ideal flow rate.

Substituting equation (9) into equation (16), the flow loss can be given by equation (18)

\[
q_{\text{loss}} = \frac{\tau(e^{\alpha T/\tau} - e^{T/\tau} + e^{(T-\alpha T)/\tau} - 1) + \alpha T(1 - \alpha)(e^{T/\tau} - 1)}{(e^{T/\tau} - 1)TR}
\]  
(18)

This shows that the flow loss is affected by the pressure difference, but not directly affected by the delivery flow rate. The flow ripple peak-to-peak amplitude is defined as the difference in the flow rates \( q_H \) and \( q_a \), which is given by equation (19)

\[
q_{\text{ripple}} = q_H - q_a = \left(\frac{(p_H - p_L)(e^{\alpha T/\tau} - e^{(T-\alpha T)/\tau} - e^{T/\tau} + 1)}{e^{T/\tau} - 1}\right)
\]  
(19)

This shows that the flow ripple is also related to the pressure difference between the high- and low-pressure ports, but not directly affected by the delivery flow rate.
Figure 5 shows the results by using equations (18), (19) and (14) to demonstrate the relationship of flow loss, flow ripple and pressure loss with different switching ratios. Parameters for the analytical model are listed in Table 1.

The resistance of the inertance tube was estimated by using equation (20)

$$R_t = \frac{128 \rho L}{\pi d^4}$$

The valve resistance was assumed to be higher than the tube resistance, in this example. A wide range of valve resistances were used in the calculation. As can be seen, the highest flow loss occurs when the switching ratio is 0.5. A large valve resistance can cause high system flow loss, but very slightly reduced flow ripple. The flow ripple of this simulated system is high, especially with the switching ratio of 0.5.

A ramped delivery flow from 0 to 2 L/s was applied to investigate the pressure loss, as shown in Figure 5 (bottom). It can be seen that the pressure loss is in direct proportion to the average delivery flow rate and system resistance. Large pressure loss can be caused by high delivery flow rate and system resistance.

Using equations (13) and (16), system efficiency can be calculated as

$$\eta = \frac{P_d \cdot q_m}{(q_{\text{loss}} + q_m \alpha)p_H + (q_m - q_{\text{loss}} - q_m \alpha)p_L}$$

$$= \frac{(p_H \alpha + p_L(1 - \alpha))q_m - q_m^2 R}{(p_H \alpha + p_L(1 - \alpha))q_m + q_{\text{loss}}(p_H - p_L)}$$

(21)

It can be seen that the efficiency is determined by the delivery flow rate, resistance, flow loss and the difference of the supply pressures.

Theoretically, for ideal operation with 100% efficiency and no losses, the input or output power is given by equations (22) and (23)

$$P_{\text{ideal}} = (p_H \alpha + p_L(1 - \alpha))q_m$$

(22)

or

$$P_{\text{ideal}} = (q_{\text{loss}} + q_m \alpha)p_H + (q_m - q_{\text{loss}} - q_m \alpha)p_L$$

(23)

where $q_{\text{loss}} = 0$

Consider the flow loss and pressure loss of the system, the actual power loss can be defined by using equation (24)

$$P_{\text{loss}} = (p_H - p_L)q_{\text{loss}} + q_m^2 R$$

(24)

Figure 6 shows system efficiency contours (in percent) and power loss (in watts). Low efficiency occurs with a very low delivery flow at different switching ratios. High power loss occurs when the switching ratio equals to 0.5. With a fixed switching ratio, a high delivery flow would cause high power loss.

Analytical characteristics for a flow booster configuration are shown in Figure 7, for a range of switching ratio $\alpha$. It can be seen that the delivery pressure is less than the supply pressure, but the high-pressure supply flow is less than the delivery flow, especially for small switching ratio $\alpha$. The efficiency is best for large ratios and delivery flows of about 0.2 L/s. The power loss is symmetrical with $\alpha$, with a peak value occurring for a ratio $\alpha = 0.5$. This also can be seen in Figure 6(b).

### Distributed parameter model

A distributed parameter model assumes that the attributes of the circuit, such as resistance, capacitance and inertance, are distributed continuously throughout the circuit. This is in contrast to the common lumped parameter model, which assumes that these values are lumped into components that are joined by perfectly conducting lines.
Distributed parameter models have been successfully used in the study of dynamic flow in pipelines.\textsuperscript{21–24}

The time-domain approach that was used for the lumped parameter model cannot readily be applied for the distributed parameter model, as a simple analytical solution cannot be obtained for the two parts of the switching cycle. Instead, a frequency-domain approach was used. The supply pressure is defined as in Figure 4, and the same assumptions are made as listed in section ‘Lumped parameter model’.

\begin{figure}[h]
\centering
\begin{subfigure}[h]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure6a}
\caption{System efficiency and power loss with switching ratio from 0 to 1: (a) system efficiency (%) and (b) system power loss (W).}
\end{subfigure}
\begin{subfigure}[h]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure6b}
\end{subfigure}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{Analytical results for a flow booster configuration based on a lumped parameter model in time domain.}
\end{figure}
Taking the Fourier series of the supply pressure \( p_s \), the Fourier coefficients \( P_n \) are, for \( n = 1 \) to \( \infty \)

\[
P_n = \frac{1}{T} \int_0^T p(t) e^{-j2\pi nt/T} dt = \frac{1}{T} \int_0^{\alpha T} p_H e^{-j2\pi nt/T} dt + \frac{1}{T} \int_0^{\alpha T} p_L e^{-j2\pi nt/T} dt + \frac{P_H}{2\pi n} (1 - e^{-j2\pi \alpha}) + \frac{P_L}{2\pi n} (e^{-j2\pi \alpha} - e^{-j2\pi})
\]

(25)

The Fourier coefficients \( Q_n \) of the flow rate can be described by using equation (26)

\[
Q_n = \frac{P_n}{Z_E}
\]

(26)

where \( Z_E \) is the entry impedance which is the ratio of the pressure ripple to the flow ripple at the entry to the hydraulic circuit.\(^{25}\)

Thus, the flow rate \( q(t) \) in time domain can be obtained by using Fourier series

\[
q(t) = 2 \sum_{n=1}^{\infty} Q_n e^{j(n2\pi t)/T} + q_m
\]

(27)

where \( q_m \) is the mean delivery flow rate.

To check the validity of the frequency-domain method, it was first applied to the lumped parameter model. In this case the entry impedance \( Z_E \) is given by equation (28)

\[
Z_E = R_e + R_f + j\omega l
\]

(28)

Figure 8 shows the comparison of using time-domain and frequency-domain methods based on the lumped parameter model. Parameters in Table 1 were re-applied in frequency-domain model. Based on Nyquist’s theorem and the timestep used in the time-domain method, 100 spectral components were sufficient in the frequency-domain model. The result from the frequency-domain approach was found to agree well with the time-domain method described in section 'Lumped parameter model'.

For the distributed parameter model, the entry impedance \( Z_E \) is given by equation (29)

\[
Z_E = jZ_0 \xi \left( \frac{\omega L \xi}{c} \right) + R_e
\]

(29)

where \( Z_0 = \rho c / A \) is the pipe characteristic impedance and \( \xi \) is the viscous wave correction factor.\(^{21}\)

Substituting equations (25) and (29) into equation (26), the Fourier coefficients \( Q_n \) are given by equation (31)

\[
Q_n = \frac{p_H}{2\pi n} (1 - e^{-j2\pi \alpha}) + \frac{p_L}{2\pi n} (e^{-j2\pi \alpha} - e^{-j2\pi})
\]

(31)

The mean high-pressure supply flow rate \( q_H \) corresponds to the integral of equation (27) through the interval \( 0 \leq t \leq \alpha T \) divided by \( T \), as shown in equation (32).

\[
\bar{q}_H = \frac{1}{T} \int_0^{\alpha T} q(t) dt
\]

(32)

Thus

\[
\bar{q}_H = -2 \sum_{n=0}^{\infty} \text{Re} \left[ \frac{Q_n}{2\pi n} (1 - e^{j2\pi \alpha}) \right] + q_m \alpha
\]

(33)

The upper limit of the summation can be chosen to give a finite number of harmonics. One hundred harmonics were considered and used for the following calculations.

Theoretically, for ideal operation the mean high pressure flow is given by

\[
\bar{q}_H = q_m \alpha
\]

(34)

Compared with equations (33) and (34), the flow loss in frequency domain can be defined as

\[
q_{loss} = -2 \sum_{n=0}^{\infty} \text{Re} \left[ \frac{Q_n}{2\pi n} (1 - e^{j2\pi \alpha}) \right]
\]

(35)

Using this expression for flow loss, the power loss can be obtained from equation (24).

For comparison, a simulation model was created using MATLAB Simulink. The TLM was used to model the wave propagation in the tube. The model was developed by Krus et al.\(^{26}\) and modified by Johnston\(^{27}\) to include unsteady or frequency-dependent friction. The TLM model accurately and efficiently
represents wave propagation and laminar friction over a very wide frequency range. However, the previous TLM models have some inherent inaccuracies. An improved alternative TLM model has been proposed by Johnston, and it was used to model the iner-tance tube in the following simulation work. A small compressible volume (5 cm³) was included between the valve model and the TLM inertance tube model.

The high-speed switching valve was assumed to switch instantaneously and was modelled with a constant resistance of 20 bar/(L/s). Figure 9 shows the results obtained by using the analytical model and simulation in one switching period.

As can be seen, the analytical and simulated results agree well. It shows that the analytical model is effective for estimating the characteristics of a SIHS, given the assumptions of instantaneous switching, linear time-invariant resistance and zero leakage. The effects of wave propagation can be seen clearly. The small differences may be due to inaccuracies in the TLM iner-tance tube model, the effect of compressible volume coupling the valve model and TLM model and numerical errors from the ODE solver. The analytical model in the frequency domain is able to estimate the dynamic flow rate and can be used for predicting system characteristics and performance.

Figure 10 shows the flow loss and peak-to-peak flow ripple of the system with different valve resistances based on a distributed parameter model. The switching frequency is 100 Hz. As in the lumped parameter model, the highest flow loss occurs with the switching ratio of 0.5 and the valve resistance of 20 bar/(L/s). However, the distributed parameter model exhibits optimal switching ratios, which result in low flow loss and power loss, which do not occur in the lumped parameter model.

Analytical characteristics for a flow booster configuration using the distributed parameter model are shown in Figure 11, for a range of switching ratio α and two switching frequencies (100 and 325 Hz). When the switching frequency is 100 Hz, the trend of efficiency and power loss are quite similar to the results obtained by using the lumped parameter model in time domain, as shown in Figure 11(a). However, the effect of wave propagation is very significant when the switching frequency of 325 Hz is applied (Figure 11(b)). It can be seen that high efficiency and low power loss occur at the switching ratio of 0.5 with a delivery flow rate below 0.5 L/s in this case. Compared with a conventional valve controlled system, assuming the required delivery pressure and flow rate were 50 bar and 0.5 L/s and the supply pressure was 100 bar which are the same operating conditions of the SIHS with the switching ratio of 0.5, the power loss would be 2500 W. This is five times of the power loss of the SIHS as shown in Figure 11. The optimized SIHS is very effective and energy efficiency.

Figure 12 shows the relationship of switching ratio, switching frequency and flow loss. The flow loss peaks occur with the switching ratio of 0.5 in a wide frequency range (0–1000 Hz), as shown in Figure 12(a). In Figure 12(b), it can be seen that large flow loss occurs at 650 Hz which is a pipeline natural frequency \( f_N = \frac{c}{2L} \) for ‘open–open’ boundary conditions (pressure prescribed at both ends).

To eliminate the high flow loss, the switching frequency should be selected below the system natural frequency. Furthermore, high flow loss also occurs when the switching frequency equals half of the natural frequency \( f = f_N/2 \) and when the switching ratio is about 0.25 or 0.75. However, optimal switching ratios or optimal frequencies can be estimated. In Figure 12(b), optimal switching frequencies, resulting in low flow loss, appear as a cross-shape with the switching frequency below 650 Hz. Since one is most likely to use a switching frequency below 300 Hz, the values of the left side of the cross curve were applied.
This corresponds to the period of the shorter of the high and low pulses being equal to the wave propagation time, \( \min (\alpha T, (1 - \alpha)T) = 2L/c \).

### Experimental results

#### Test rig

A flow booster configuration was constructed as the test rig, as shown in Figure 13. The system mainly comprised a proportional valve, an inerance tube comprising two rigid tubes connected in series and a loading system comprising a pressure compensated valve and a needle valve. Experiments were carried out to verify the analytical model.

The hydraulic power pack with a maximum supply pressure of 300 bar was applied as the high-pressure supply source, and the low supply pressure was controlled by the pressure reducing valve. Three accumulators and in-line shock suppressors were used to maintain constant pressures at the high-pressure supply, low-pressure supply and load. The initial charging pressures of the accumulators were 60, 20 and 35 bar, respectively, and half charging pressures of the accumulators 30, 10 and 17.5 bar were applied to the shock suppressors.

The proportional directional valve series DFplus from Parker Hannifin was used as a high-speed...
switching valve. To ensure a short switching time, valve demands of 20% opening were used instead of 100% opening. The switching time was approximately 2 ms. However, a large valve resistance is expected when only 20% open was applied. The inertance tube consisted of two pipe sections with lengths of 4.8 and 3.1 m, with a pressure transducer between, arranged in a large loop of approximately 1 m diameter. This is significantly longer than would be used in a practical application because the switching valve used has a slower response than desired for this investigation. In more realistic applications, the switching frequency might be over 100 Hz and the inertance tube length of the order of 1 m, but this would require a higher performance valve. However, the valve used here was sufficient for proof of principle.

Three miniature piezoresistive pressure transducers (Measurement Specialties EPX series EPX-N03) with a pressure range of 100 bar were used to measure the supply, upstream pressure between the switching valve and the tube and the mid-stream pressure at the connection. The other one with a pressure range of 35 bar was used to measure the low supply pressure. The downstream pressure was measured by Druck high-performance transducer PDCR series with a pressure range of 100 bar. Two gear flow meters were applied for the measurement of upstream flow rates. The one is with a flow rate range from 0.1 to 7 L/min which was connected to the low supply pressure port, while the other one with a range of 0.5–70 L/min was used to measure the flow rate from the high supply pressure port.

Parameters for the analytical model and experiments are listed in Table 2.

### Table 2. Parameters for the analytical model and experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Density, ( \rho )</td>
<td>860 kg/m³</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
<td>38 cSt</td>
</tr>
<tr>
<td>Switching frequency, ( f )</td>
<td>20/40 Hz</td>
</tr>
<tr>
<td>Inertance tube length, ( L )</td>
<td>7.9 m</td>
</tr>
<tr>
<td>Inertance tube diameter, ( d )</td>
<td>7 mm</td>
</tr>
<tr>
<td>Speed of sound, ( c )</td>
<td>1350 m/s</td>
</tr>
<tr>
<td>Oil temperature, ( C )</td>
<td>40 °C</td>
</tr>
</tbody>
</table>

Figure 13. Schematic of the test rig in a flow booster configuration.

Figure 14. Relationship of the delivery pressure and switching ratio with delivery flow rate of 0 L/min.

**Delivery pressure**

First, the relationship between delivery pressure, supply pressures and delivery flow rate was validated in experiments. As equation (13) shows, theoretically the delivery pressure is a linear function of the high and low supply pressures, the switching ratio, the delivery flow rate and the system resistance \( R \). The delivery pressure is independent of the switching frequency.

The high supply pressure was fixed at 62 bar and the low pressure at 23 bar. The needle valve was fully closed to ensure no flow rate through the system, in order to eliminate the effect of the resistance \( R \) on the load pressure. Figure 14 shows the relationship between the delivery pressure and the switching ratio with switching frequencies of 20 and 40 Hz, respectively. As can be seen, the analytical and experimental results agree very well and a clear linear characteristic of the delivery pressure is shown. The results with different switching frequencies are very similar, which indicates that the delivery pressure is not affected significantly by the switching frequency.

The effects of the delivery flow rate and the system resistance were investigated. The delivery flow rate was...
set to 6 L/min, which is expected to result in a constant pressure loss occurred between the system upstream supply and downstream delivery ports, as defined in equations (13) and (14), provided that the overall system resistance is reasonably constant. The delivery pressure and pressure loss are shown in Figure 15, where about 12 bar constant pressure drop occurred. The system overall resistance ($R = 120 \text{ bar/(L/s)}$) can be estimated by using the pressure drop of 12 bar and the delivery flow rate of 6 L/min. This is mainly due to the resistance of the switching valve, resistance of the long inertance tube, the leakage and the switching transition of the switching valve. A shorter inertance tube and a higher performance switching valve can improve system efficiency and decrease the pressure loss.

**Flow loss and power loss**

The flow loss is strongly related to the effect of wave propagation when the device switches. Low flow loss occurs when the switching frequency and switching ratio satisfy equation (36). Figure 16 shows the flow loss versus the switching ratio with a switching frequency of 40 Hz for the loading flow rate of 0 and 6 L/min, respectively. With the system overall resistance of 120 bar/(L/s) and the tube resistance of about 38 bar/(L/s), the valve resistance $R_v$ can be estimated as 82 bar/(L/s), which is used in the analytical model. It can be seen that the flow loss curves are generally symmetrical and good agreement between the analytical and experimental results is obtained. Also, the optimal switching ratio of 0.5 is shown clearly in the experimental and analytical curves.

Figure 17 shows the experimental and analytical power losses of the system. As can be seen, the larger delivery flow rate results in a higher system power loss. A lower power loss occurs at the optimal switching frequency. Figure 18 shows a comparison of the experimental and analytical efficiency of the SIHS with a constant delivery flow rate of 6 L/min. The system efficiency increased with an increasing switching ratio. Compared with a conventional valve controlled system, assuming the required delivery pressure and flow rate were 30 bar and 6 L/min and the supply pressure was 62 bar which are the same operating conditions of the SIHS with the switching ratio of 0.5, the power loss is 320 W and the efficiency is 48.4%. This is roughly twice the power loss of the SIHS as shown in Figure 17 and 20% lower efficiency than the SIHS as shown in Figure 18. However, the test rig is actually not an ideal design system for high efficiency. The switching valve and inertance tube used for experiments were not optimized but sufficient for proof of principle of the SIHS and verifying the proposed analytical model. A more efficient test rig with a high-speed switching valve and a short inertance tube will be studied in future work.
Discussion

Analytical models for investigating the SIHS based on a lumped parameter model and a distributed parameter model have been found to agree well with numerical simulations and experimental results. Time-domain numerical simulations have been found to be very slow to run because of the fast switching behaviour and the need for high bandwidth, high fidelity pipeline models, whereas the analytical models are fast and efficient. The analytical models are very effective tools for understanding and analysing the characteristics and performance of the SIHS, such as given below.

- System pressure and flow rate characteristics.
- System flow loss, pressure loss and flow ripple.
- System power loss and efficiency.

Because of their high computational speed, they can also be used to optimize the design and operating conditions of the SIHS.

However, some limitations exist by using this analytical model, such as given below.

- The switching transition is assumed to be instantaneous and the valve resistance is assumed to be constant. The practical system power loss is expected to be higher than the analytical calculation.
- Valve leakage and charge loss of the accumulators are neglected. These could result in an additional power loss as well.

The valve closure and opening during the switching transition could be modelled as a transient increase in the valve resistance. This is difficult to implement in the time-domain and frequency-domain analytical models presented here. Valve leakage is also difficult to implement in these models. A method to include non-instantaneous switching transition and leakage in the analytical model is being developed and will be presented in a later article.

Conclusion

The analytical results show a very promising SIHS performance. The main advantage of the SIHS is that the inherent reactive behaviour of hydraulic tube is effectively used to control flow and pressure instead of relying on dissipation of power as in traditional throttling valve-based systems. Using the SIHS, the system efficiency is expected to be increased, while the energy cost is decreased. However, pulsations and noise may be a significant problem.

The analytical and simulated results show that the proposed analytical models are effective for investigating and analysing the characteristics and performance of a SIHS. The analytical results based on a lumped parameter model and a distributed parameter model agree well with time-domain numerical simulation results. The analytical models have been found to be reliable and effective and are much faster to run than the time-domain numerical simulations. Parameter optimization of the SIHS, such as the tube length, diameter, switching frequency and ratio, can be studied and estimated by using this model. The analytical models have been validated in experiments, and the results on a test rig show a very promising performance. There is considerable work still to be done to investigate thoroughly the performance of a SIHS including the consideration of the losses that occur in the switching transition period.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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References

Appendix

Notation

- \( c \) speed of sound
- \( d \) tube internal diameter
- \( f_s \) switching frequency
- \( f_n \) natural frequency
- \( I \) tube inertia
- \( j \) imaginary unit
- \( J_0, J_1 \) Bessel functions of the first kind of orders 0 and 1
- \( L \) tube length
- \( n \) number of harmonics
- \( p_d \) delivery pressure
- \( p_i \) inlet pressure
- \( p_H \) high supply pressure
- \( p_{\text{loss}} \) pressure loss
- \( p_L \) low supply pressure
- \( P_{\text{ideal}} \) ideal system power
- \( P_{\text{loss}} \) system power loss
- \( P_n \) Fourier coefficient of pressure
- \( q \) flow rate
- \( q_d \) initial flow rate during the rising phase
- \( q_b \) initial flow rate during the falling phase
- \( q_H \) steady-state flow rate from the high-pressure supply port

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{q}_H$</td>
<td>average flow rate from the high-pressure supply port</td>
<td>$T$</td>
<td>switching cycle</td>
</tr>
<tr>
<td>$q_L$</td>
<td>steady-state flow rate from the low-pressure supply port</td>
<td>$V_{ab}$</td>
<td>flow volume during the period $0 \leq t \leq \alpha T$</td>
</tr>
<tr>
<td>$q_{loss}$</td>
<td>flow loss</td>
<td>$V_{hc}$</td>
<td>flow volume during the period $\alpha T &lt; t \leq T$</td>
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<tr>
<td>$\bar{q}_L$</td>
<td>average flow rate from the low-pressure supply port</td>
<td>$Z_0$</td>
<td>pipe characteristic impedance</td>
</tr>
<tr>
<td>$q_m$</td>
<td>average delivery flow rate</td>
<td>$Z_E$</td>
<td>entry impedance</td>
</tr>
<tr>
<td>$q_{ripple}$</td>
<td>flow ripple</td>
<td>$\alpha$</td>
<td>switching ratio</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>Fourier coefficient of flow rate</td>
<td>$\Delta \rho$</td>
<td>pressure drop</td>
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<tr>
<td>$R$</td>
<td>overall resistance of the system</td>
<td>$\eta$</td>
<td>system efficiency</td>
</tr>
<tr>
<td>$R_t$</td>
<td>resistance of the inertance tube</td>
<td>$\nu$</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>$R_v$</td>
<td>resistance of the high-speed switching valve</td>
<td>$\xi$</td>
<td>viscous wave correction factor</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time constant</td>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\omega$</td>
<td>radian frequency</td>
<td>$\tau$</td>
<td>time constant</td>
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